

RETEACHING 4-1

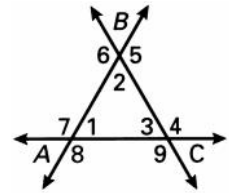
TRIANGLES AND TRIANGLE THEOREMS

Interior angles: $\angle 1, \angle 2, \angle 3$

Exterior angles: $\angle 4, \angle 5, \angle 6, \angle 7, \angle 8, \angle 9$

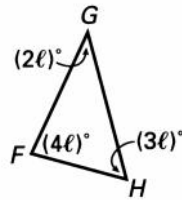
According to the triangle-sum theorem,
 $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$.

According to the exterior angle theorem,
 $m\angle 9 = m\angle 1 + m\angle 2$.



Example 1

Refer to the triangle at the right. Find $m\angle F$.



Solution

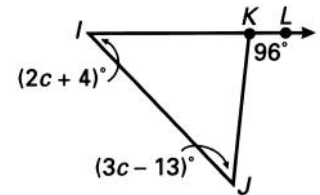
Use the triangle-sum theorem to write and solve an equation.

$$\begin{aligned} m\angle F + m\angle G + m\angle H &= 180 \\ 4l + 2l + 3l &= 180 \\ 9l &= 180 \\ l &= 20 \end{aligned}$$

So, the value of l is 20. From the figure, $m\angle F = (4l)^\circ$. Substituting 20 for l , $m\angle F = (4 \cdot 20)^\circ = 80^\circ$.

Example 2

Refer to the triangle at the right. Find $m\angle KIJ$.



Solution

Use the exterior angle theorem to write and solve an equation.

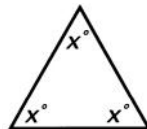
$$\begin{aligned} m\angle KIJ + m\angle IJK &= m\angle LKJ \\ (2c + 4) + (3c - 13) &= 96 \\ 5c - 9 &= 96 \\ 5c &= 105 \\ c &= 21 \end{aligned}$$

So the value of c is 21. From the figure, $m\angle KIJ = (2c + 4)^\circ$. Substituting 21 for c , $m\angle KIJ = (2 \cdot 21 + 4)^\circ = (42 + 4)^\circ = 46^\circ$.

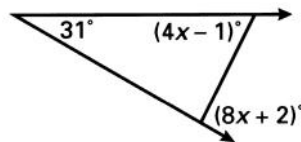
EXERCISES

Find the value of x in each figure.

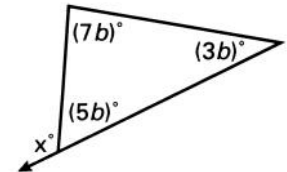
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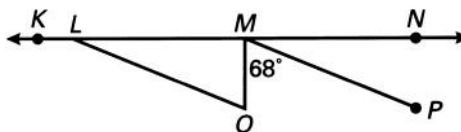
2.



3.



In the figure at the right, $\overline{MO} \perp \overline{KN}$ and $\overline{LO} \parallel \overline{MP}$. Find the measure of each angle.



4. $\angle LMO$ _____

5. $\angle OLM$ _____

6. $\angle LOM$ _____

7. $\angle KLO$ _____

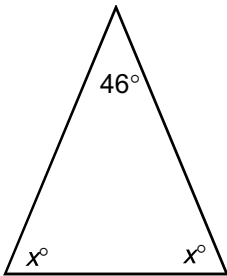
EXTRA PRACTICE **4-1**

TRIANGLES AND TRIANGLE THEOREM

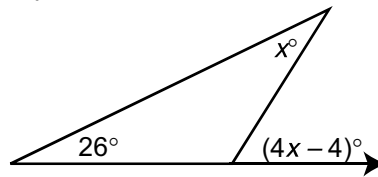
EXERCISES

Find the value of x in each figure.

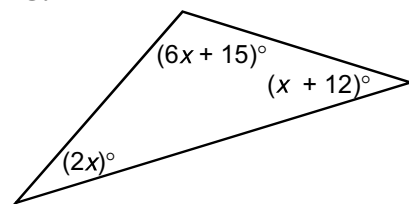
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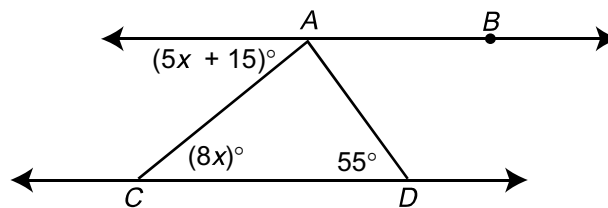


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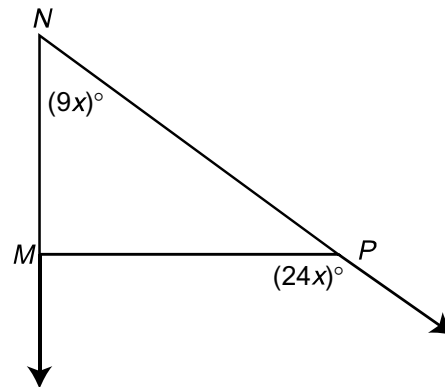
4. In the figure at the right, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

Find $m\angle ACD$. _____



5. In the figure at the right, $\overleftrightarrow{NM} \perp \overleftrightarrow{PM}$.

Find $m\angle MNP$. _____



6. In $\triangle RST$, $m\angle R$ is 20° more than two times $m\angle S$, and $m\angle S = m\angle T$. Find the measure of each angle. _____

7. The measure of the smallest angle of a triangle is 34° less than the measure of the largest angle of the triangle. The measure of the third angle is 14° more than the measure of the smallest angle. Find all three measures. _____

On a coordinate plane on your own paper, sketch the triangle with the given vertices. Then classify the triangle, first by its sides, then by its angles.

8. $A(-2, -2)$, $B(0, 4)$, $C(2, -2)$

9. $R(-5, 1)$, $S(-2, 3)$, $T(6, 0)$

RETEACHING 4-6**INEQUALITIES IN TRIANGLES**

If two sides of a triangle are unequal in length, then the angle opposite the longer side has the greater measure. If two angles of a triangle are unequal in measure, then the side opposite the angle with the greater measure has the longer length.

Example 1

Two sides of a triangle measure $6\frac{1}{2}$ in. and $3\frac{1}{2}$ in. Find the range of lengths for the third side.

Solution

Use the variable l to represent the length in inches of the third side. By the triangle inequality theorem, these three inequalities must be true.

$$\begin{aligned} \text{a. } 6\frac{1}{2} + 3\frac{1}{2} &> l \\ 10 &> l \end{aligned}$$

$$\begin{aligned} \text{b. } 6\frac{1}{2} + l &> 3\frac{1}{2} \\ l &> -3 \end{aligned}$$

$$\begin{aligned} \text{c. } 3\frac{1}{2} + l &> 6\frac{1}{2} \\ l &> 3 \end{aligned}$$

Inequality b is not useful since a length must be a positive number. Combining inequalities a and c results in the inequality $10 > l > 3$, so the third side must be less than 10 in. and greater than 3 in.

Example 2

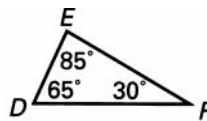
Refer to $\triangle DEF$. List the sides from longest to shortest.

Solution

List the angles from the largest to smallest:

$m\angle E > m\angle D > m\angle F$. By the unequal angles theorem, opposite sides of unequal angles will be unequal in length in the same order.

So, $DF > EF > DE$.

**EXERCISES**

The lengths of two sides of a triangle are given. Find the range of lengths for the third side.

1. 3 mm, 5 mm

2. 1.2 in., 4.6 in.

3. 2.5 cm, 8 cm

4. In $\triangle GHI$, $GH = 3$ cm, $HI = 4$ cm, and $GI = 3.5$ cm.

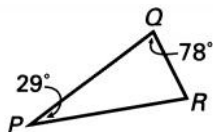
List the angles of the triangle in order from smallest to largest. _____

5. In $\triangle MNO$, $m\angle M > m\angle O$ and $m\angle N < m\angle O$.

List the sides of the triangle in order from shortest to longest. _____

Which side is the longest side of each triangle? Which side is the shortest?

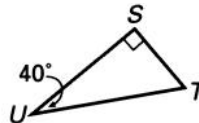
6.



Longest _____

Shortest _____

7.



Longest _____

Shortest _____

EXTRA PRACTICE 4-6

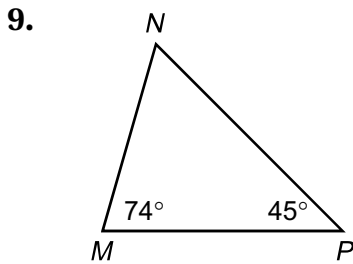
INEQUALITIES IN TRIANGLES

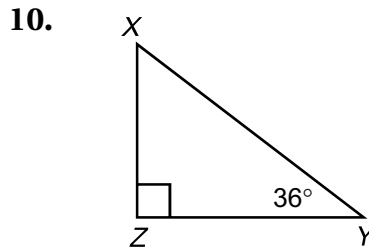
EXERCISES

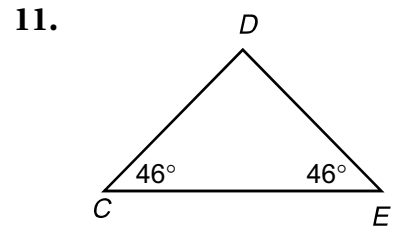
Can the given measures be the lengths of the sides of a triangle?

- | | |
|---|---------------------------------|
| 1. 4 cm, 5 cm, 6 cm _____ | 2. 9.1 m, 5.6 m, 7.5 m _____ |
| 3. 15 in., 24 in., 19 in. _____ | 4. 8 cm, 12 mm, 4 cm _____ |
| 5. $5\frac{3}{4}$ in., 9 in., $12\frac{1}{8}$ in. _____ | 6. 3.5 yd, 3.5 yd, 3.5 yd _____ |
| 7. 4 ft, 3 yd, 6 ft _____ | 8. 5 m, 0.5 km, 3 m _____ |

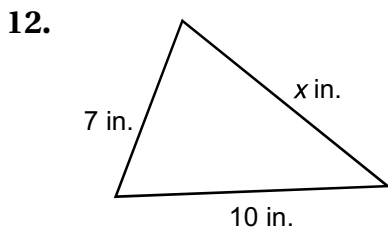
Which is the longest side of each triangle? the shortest?

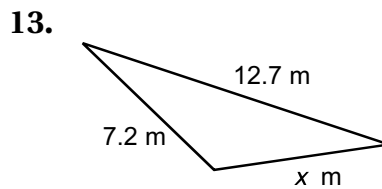


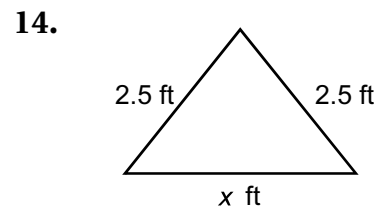




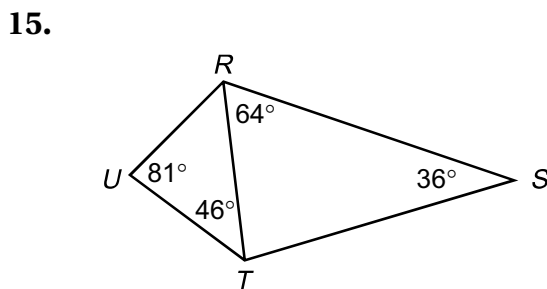
In each figure, give the range of possible values for x .

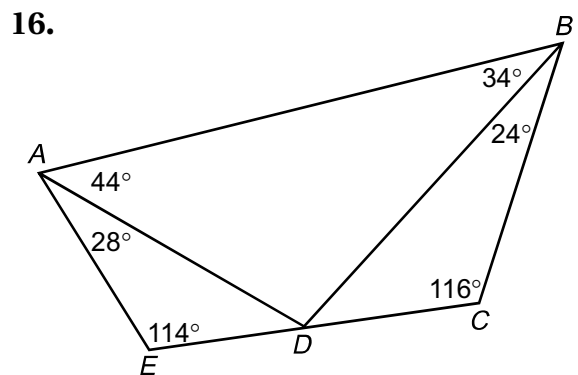






List all the segments in each figure *in order* from longest to shortest.

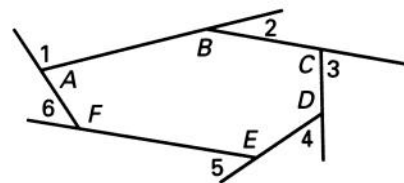




RETEACHING 4-7

POLYGONS AND ANGLES

Polygons are closed plane figures formed by joining three or more coplanar segments at their endpoints. A polygon is convex when each line containing a side contains no points in the interior of the polygon. The sum of the measures of the angles of a convex polygon with n sides can be found by solving the equation $(n - 2)(180)^\circ$. The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360° .



$$m\angle A + m\angle B + m\angle C + m\angle D + m\angle E + m\angle F = (n - 2)180^\circ \text{ or } (6 - 2)180^\circ = (4)180^\circ = 720^\circ$$

Exterior angles: $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6$
 $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 = 360^\circ$

Example

- a. Find the measure of each interior angle of a regular pentagon.
- b. Find the measure of each exterior angle of a regular pentagon.

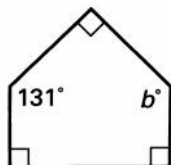
Solution

- a. A pentagon has five sides. Use the polygon-sum theorem to find the sum of the measures of the interior angles.
 $(n - 2)180^\circ = (5 - 2)180^\circ = (3)180^\circ = 540^\circ$
 Because the pentagon is regular, each interior angle is equal in measure. So, the measure of one interior angle is $540^\circ \div 5 = 108^\circ$.
- b. By the polygon exterior angle theorem, the sum of the measures of the exterior angles is 360° . Because the pentagon is regular, each exterior angle is equal in measure. So, the measure of one exterior angle is $360^\circ \div 5 = 72^\circ$.

EXERCISES

Find the unknown angle measure or measures in each figure.

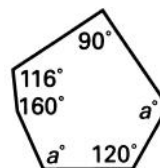
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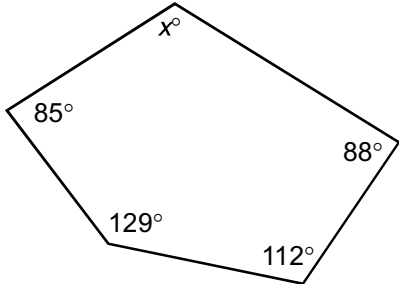


- 5. Find the measure of each interior angle of a regular decagon. _____
- 6. Find the measure of each exterior angle of a regular hexagon. _____
- 7. Find the sum of the measures of the interior angles of a polygon with 22 sides. _____

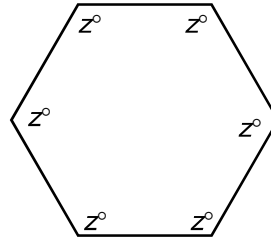
EXTRA PRACTICE **4-7****POLYGONS AND ANGLES****EXERCISES**

Find the unknown angle measure in each figure.

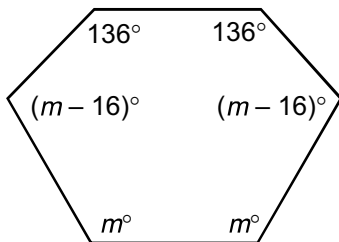
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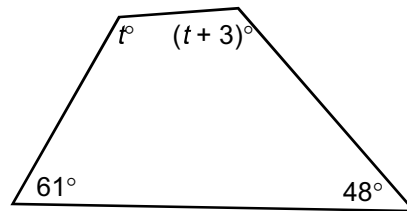
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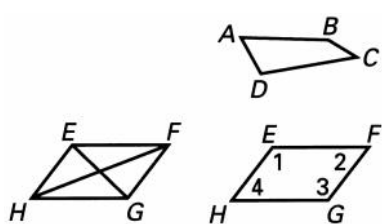
5. Find the measure of each interior angle of a regular decagon. _____
6. Find the sum of the measures of the interior angles of a regular polygon with 20 sides. _____
7. Find the sum of the measures of the exterior angles of a regular heptagon. _____
8. Find the measure of each exterior angle of a regular polygon with 30 sides. _____
9. Find the measure of each interior angle of a regular polygon with 24 sides. _____
10. Find the sum of the measures of the interior angles of a regular polygon with 30 sides. _____

Find the number of sides of each regular polygon.

11. The measure of each exterior angle is 4° . _____
12. The sum of the measures of the interior angles is 6120° . _____
13. The measure of each interior angle is 160° . _____
14. The sum of the measures of the interior angles is 2520° . _____
15. The measure of each interior angle is 170° . _____

RETEACHING 4-8

SPECIAL QUADRILATERALS: PARALLELOGRAMS

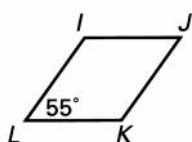


Quadrilateral
Parallelogram

Opposite sides: \overline{AB} and \overline{CD} , \overline{BC} and \overline{DA}
 Opposite angles: $\angle A$ and $\angle C$, $\angle B$ and $\angle D$
 $\overline{EF} \parallel \overline{HG}$ $m\angle 1 = m\angle 3$
 $\overline{EH} \parallel \overline{FG}$ $m\angle 2 = m\angle 4$
 \overline{HF} bisects \overline{EG} , and \overline{EG} bisects \overline{HF} .
 $\overline{EF} = \overline{HG}$

Example 1

Refer to $\square IJKL$ at the right.
 Find $m\angle I$ in $\square IJKL$.



Solution

Since $\angle L$ and $\angle J$ are opposite angles, by the parallelogram-angle theorem,
 $m\angle L = m\angle J = 55^\circ$.

Use the polygon-sum theorem to find the sum of the measures of the interior angles.

$(n - 2)180^\circ = (4 - 2)180^\circ = (2)180^\circ = 360^\circ$
 Notice that $\angle L + \angle J = 55^\circ + 55^\circ = 110^\circ$.

It follows that

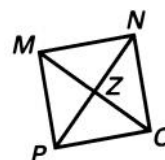
$m\angle I + m\angle K = 360^\circ - 110^\circ = 250^\circ$.

Since $\angle I$ and $\angle K$ are opposite angles, by the parallelogram-angle theorem,

$m\angle I = m\angle K = 250^\circ \div 2 = 125^\circ$.

Example 2

In the figure at the right, $MNOP$ is a square and $PN = 9$ cm. Find OZ .



Solution

A square is also a rectangle. By the rectangle-diagonal theorem, the diagonals are equal in length. So,
 $PN = OM = 9$ cm.

By the parallelogram-diagonal theorem, the diagonals bisect each other.

So, $OM = OZ + MZ = 2(OZ)$.

Substitute 9 for OM and solve.

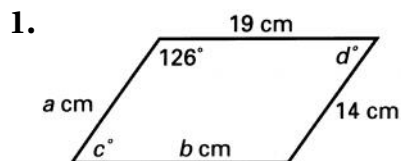
$2OZ = 9$

$OZ = 4.5$

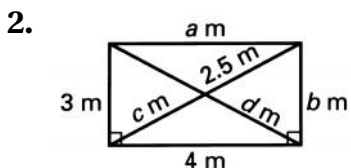
The length of \overline{OZ} is 4.5 cm.

EXERCISES

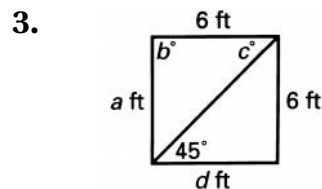
Find the values of a , b , c , and d in these parallelograms.



$a = \underline{\hspace{2cm}}$ $b = \underline{\hspace{2cm}}$
 $c = \underline{\hspace{2cm}}$ $d = \underline{\hspace{2cm}}$

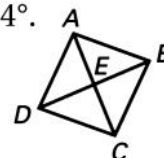


$a = \underline{\hspace{2cm}}$ $b = \underline{\hspace{2cm}}$
 $c = \underline{\hspace{2cm}}$ $d = \underline{\hspace{2cm}}$



$a = \underline{\hspace{2cm}}$ $b = \underline{\hspace{2cm}}$
 $c = \underline{\hspace{2cm}}$ $d = \underline{\hspace{2cm}}$

Quadrilateral $ABCD$, shown at the right, is a rhombus with $m\angle ABC = 84^\circ$.



- 4. $\angle DAB$ _____
- 5. $\angle AED$ _____
- 6. $\angle ADB$ _____
- 7. $\angle DCE$ _____

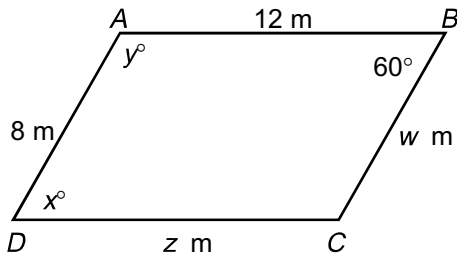
EXTRA PRACTICE **4-8**

SPECIAL QUADRILATERALS: PARALLELOGRAMS

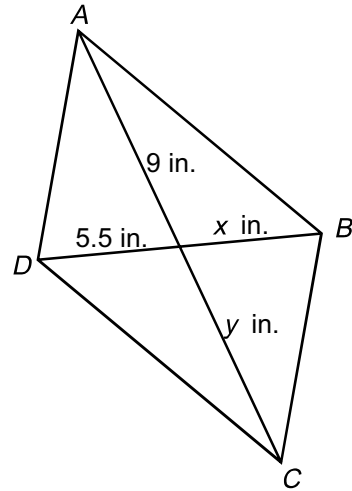
EXERCISES

In Exercises 1–4, $ABCD$ is a parallelogram. Find the values of x , y , z , and w .

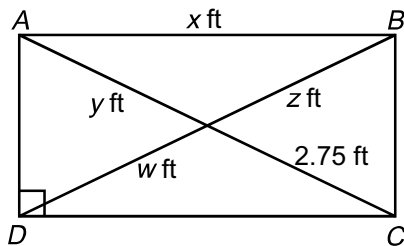
1.



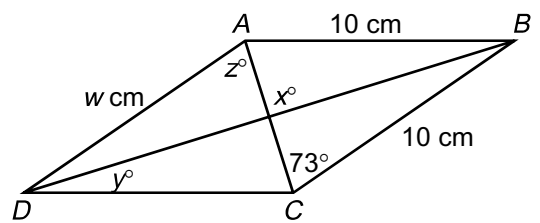
2.



3.



4.



Tell whether each statement is *true* or *false*.

5. All parallelograms are rectangles. _____

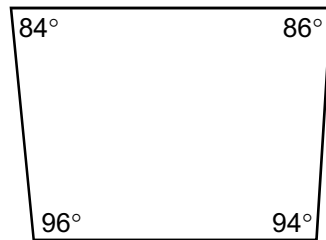
6. No rectangles are squares. _____

Do you think that the given figure is a parallelogram? Write *yes* or *no*. Explain.

7.

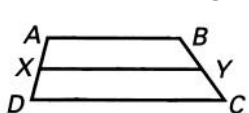


8.



RETEACHING 4-9

SPECIAL QUADRILATERALS: TRAPEZOIDS



Bases: \overline{AB} and \overline{DC}

Base angles: $\angle A$ and $\angle B$; $\angle D$ and $\angle C$

$\overline{AB} \parallel \overline{DC}$

Legs: \overline{AD} and \overline{BC}

Median: \overline{XY}

An isosceles trapezoid has legs of equal length and base angles with equal measures.

Example

In the figure at the right, $\overline{EH} \parallel \overline{FG}$, $\overline{EH} \parallel \overline{XY}$, and $EF = HG$.

- a. Find XY . b. Find $m\angle G$.

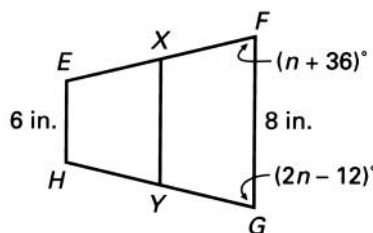
Solution

Quadrilateral $EFGH$ is an isosceles trapezoid with bases \overline{EH} and \overline{FG} .

- a. Since \overline{XY} is the median, apply the trapezoid-median theorem.

$$\begin{aligned} XY &= \frac{1}{2}(EH + FG) \\ &= \frac{1}{2}(6 + 8) \\ &= \frac{1}{2}(14) \\ &= 7 \end{aligned}$$

So, the length of \overline{XY} is 7 in.



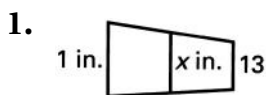
- b. By the isosceles-trapezoid theorem, the base angles $\angle F$ and $\angle G$ are equal in measure. Use this fact to write and solve an equation.

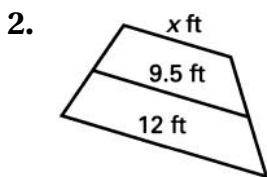
$$\begin{aligned} n + 36 &= 2n - 12 \\ n + 36 - n &= 2n - 12 - n \\ 36 &= n - 12 \\ 48 &= n \end{aligned}$$

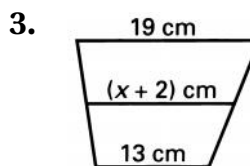
So, the value of n is 48. Use $m\angle G = (2n - 12)^\circ$. Substituting 48 for n , $m\angle G = (2 \cdot 48 - 12)^\circ = (96 - 12)^\circ = 84^\circ$.

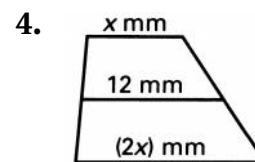
EXERCISES

A trapezoid and its median are shown. Find the value of x .

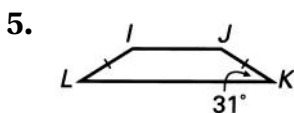


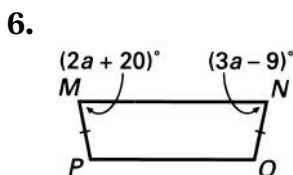


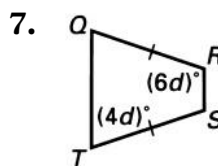


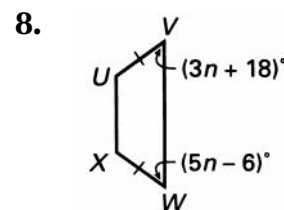


The given figure is a trapezoid. Find the measures of all the unknown angles.







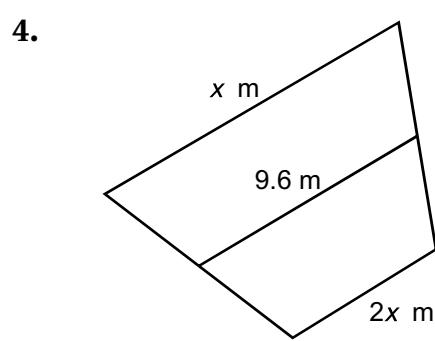
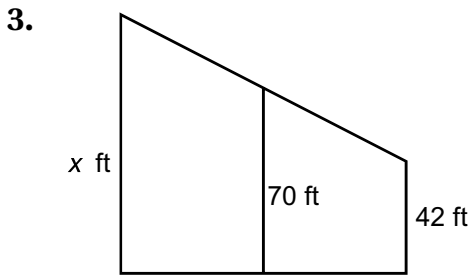
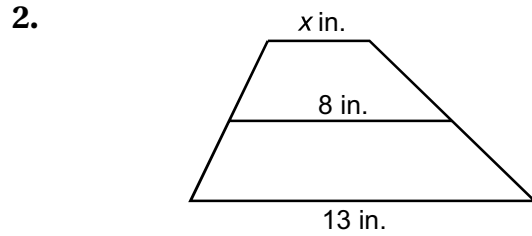
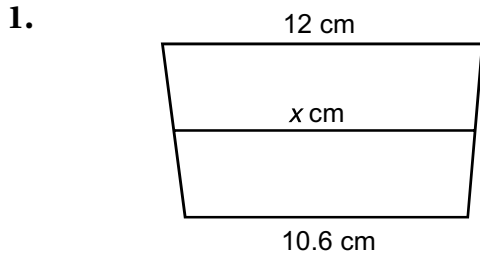


EXTRA PRACTICE **4-9**

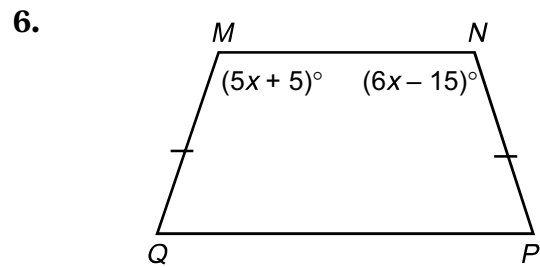
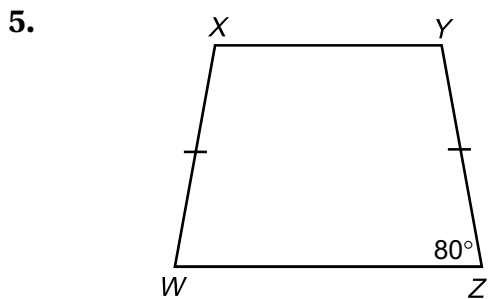
SPECIAL QUADRILATERALS: TRAPEZOIDS

EXERCISES

A trapezoid and its median are shown. Find the value of x .



The given figure is a trapezoid. Find all the unknown angle measures.



CHAPTER 4

TRIANGLES, QUADRILATERALS, AND OTHER POLYGONS

STANDARDIZED TEST PRACTICE

1. Which is not a rational number?

- A. $\sqrt{81}$ B. $-\frac{1}{9}$
 C. 0.7 D. $\sqrt{3}$
 E. $0.\overline{6}$

2. Evaluate $-2a - 3b$ if $a = -5$, $b = -3$.

- A. 1
 B. -19
 C. -1
 D. 13
 E. 19

3. Simplify. $x^5 \cdot x^3$

- A. x^8 B. x^{15}
 C. x^2 D. $2x^{15}$
 E. none of these

4. Evaluate x^{-2} if $x = 5$.

- A. 25
 B. $-\frac{1}{25}$
 C. $\frac{1}{25}$
 D. -25
 E. none of these

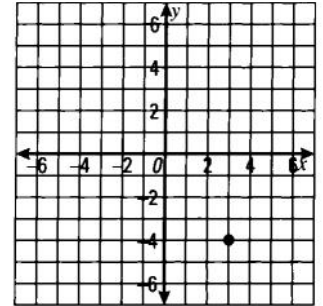
Name _____

Date _____

Scoring Record	
Possible: 20	Earned:

5. Which point is shown on the graph?

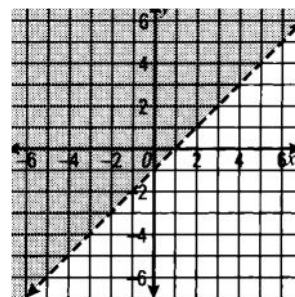
- A. (3, -4)
 B. (4, -3)
 C. (-3, -4)
 D. (-4, -3)
 E. (3, 4)



6. What is the solution for $x + 5 - 2 = |-8|$?

- A. -5 B. 5
 C. -11 D. 11
 E. none of these

7. Which inequality is shown on the graph?



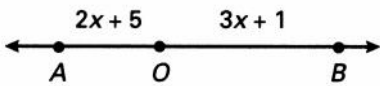
- A. $y \leq x - 1$ B. $y < x - 1$
 C. $y > x - 1$ D. $y \geq x - 1$
 E. $y > x$

8. Find the mean of the following data.

37 49 56 37 91

- A. 37
- B. 49
- C. 45
- D. 56
- E. none of these

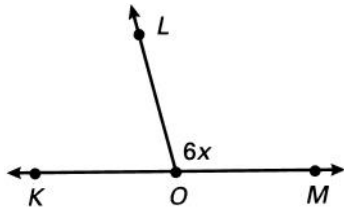
9. In the figure $AB = 91$, find AO .



- A. 39
- B. 58
- C. 19
- D. 11
- E. none of these

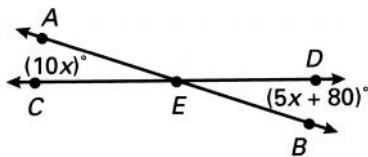
10. What is the $m\angle LOM$ if $m\angle KOL$ is $(90 - x)^\circ$?

- A. 18°
- B. 72°
- C. 54°
- D. 108°
- E. 84°



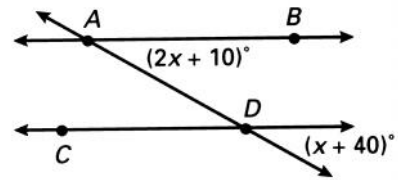
11. In the figure, \overleftrightarrow{AB} and \overleftrightarrow{CD} intersect at E . Find $m\angle AEC$.

- A. 16°
- B. 160°
- C. 80°
- D. 20°
- E. none of these



12. In the figure, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$. Find $m\angle ADC$.

- A. 70°
- B. 110°
- C. 150°
- D. 30°
- E. 15°



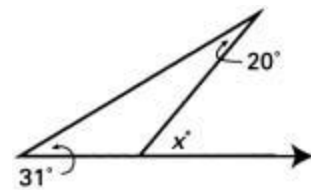
13. Which statement is the converse of the following?

If $x + y = 16$, then $x = 16 - y$.

- A. If $x + y \neq 16$, then $x \neq 16 - y$.
- B. If $x = 16 - y$, then $x + y = 16$.
- C. If $x + y = 16$, then $x = -y + 16$.
- D. If $x = 16 - y$, then $x + y \neq 16$.
- E. none of these

14. Find the value of x in the figure.

- A. 11°
- B. 129°
- C. 51°
- D. 169°
- E. 39°



15. If a polygon has 6 sides, find the sum of the interior angles.

- A. 540°
- B. 720°
- C. 1080°
- D. 2160°
- E. none of these